Appendix: Mathematical Formulas

The examination of a text that contains an astronomical or an astrological component may sometimes call for the verification of the values it presents or implies. In what follows I present some of the formulas that are required in such cases. I have used a continuous example for Formulas 1–8.

A word of warning: minor differences from the values presented in my example should not be regarded as cause for alarm. They will (if minor) be the result of differences in rounding, as between four-figure and seven-figure tables, for instance.

Symbols:

terrestrial latitude	P(hi)
celestial longitude	L(ambda)
obliquity of the ecliptic	E(psilon)
right ascension	RA
oblique ascension	OA
declination	D(elta)
semi-diurnal arc	H(alf)D(ay)
circle of position	CP
azimuth	AZ
altitude	AL

1. To find the right ascension of the meridian (RA_{10}).

(The purpose here will often be to convert a celestial longitude to a right ascension, in order to make a calculation, and then to convert back to a new celestial longitude.)

Formula: $\tan RA_{10} = \tan L_{10} \cos E$

Example: to find RA_{10} when L_{10} is Pisces 5:

$$\begin{aligned} \tan RA_{10} &= \tan 335^\circ \cos 23.525^\circ \\ &= -0.4275 \\ RA_{10} &= 336.8508^\circ \end{aligned}$$

This calculation relates to bodies located on the ecliptic; their celestial latitude is therefore 0°, and disregarded.

2. To find the declination of the sun (D).

Formula: *either,* $\sin D = \sin E \sin L$
or, $\cos D = (\cos L \cos RA) + (\sin L \sin RA \cos E)$

Example: to find D when L = Pisces 5:

$$\begin{aligned}\cos D &= (\cos 335^\circ \cos 336.85^\circ) \\ &\quad + (\sin 335^\circ \sin 336.85^\circ \cos 23.525^\circ) \\ &= 0.9857 \\ D &= -9.7115^\circ\end{aligned}$$

Declinations are positive between Aries and Virgo, negative between Libra and Pisces.

3. To find the semidiurnal arc (HD).

Formula: $\cos HD = -\tan P \tan D.$

Example: to find HD when L = Pisces 5:

$$\begin{aligned}\cos HD &= -\tan 53^\circ \tan -9.7115^\circ \\ &= 0.2271 \\ &= 76.8731^\circ.\end{aligned}$$

Convert degrees to time: = 5 hrs 7 mins 29 secs.

In other words, at the given terrestrial latitude, the day will be 10 hrs 15 mins long when the sun is at Pisces 5.

4. To find the duration of dawn (d):

Formula: $d = y - HD$, where $\cos y = \dfrac{\cos 108^\circ - \sin P \sin D}{\cos P \cos D}$

Dawn is conventionally taken to begin when the sun comes to be 18° below the horizon: 90° + 18° = 108°.

Example: to find d when L = Pisces 5:

$$\begin{aligned}\cos y &= \frac{\cos 108^\circ - \sin 53^\circ \sin -9.7115^\circ}{\cos 53^\circ \cos -9.7115^\circ} \\ &= -0.2938 \\ y &= 107.0874^\circ\end{aligned}$$

$$\begin{aligned}y - HD &= 107.0874^\circ - 76.8731^\circ \\ &= 30.2143^\circ.\end{aligned}$$

Convert degrees to time: = 2 hrs 0 mins 52 secs.

5. To find the oblique ascension of the horizon (OA_h).

Formula: $OA_h = RA_h + (90 - HD)$

Example: to find OA_h when L_h = Pisces 5:

$$\begin{aligned} OA_h &= 336.8508^\circ + (90^\circ - 76.8731^\circ) \\ &= 349.9777^\circ \end{aligned}$$

OA is less than RA between Aries and Virgo, greater between Libra and Pisces.

6. To find the astrological houses.

I describe two methods: the first is the one in common use in the middle ages, the second the one in common use in the Renaissance.

6.1 The Right Ascension Method (Alchabitius):

let	$(RA_1 - RA_{10})/3$ be x;
then	$RA_{10} + x = RA_{11}$
	$RA_{10} + 2x = RA_{12}$
and	$RA_{11} + 120^\circ = RA_3$
	$RA_{12} + 60^\circ = RA_2$

Convert RA to longitudes by adapting Formula 1

Example: to find the cusps of the houses when L_{10} is Pisces 5:
1. If L_{10} is 335°, RA_{10} is 336.8508° (Formula 1).
2. L_1 is then found by the following:

$$\cot L_1 = \tan(-RA_{10}) \cos E - \frac{\sin E \tan P}{\cos RA_{10}}$$

$$L_1 = 100.4289^\circ.$$

3. Hence, $RA_1 = 101.3508^\circ$ (Formula 1).
4. $(RA_1 - RA_{10})/3 = 41.50^\circ$.
5. Therefore:

$RA_{10} = 336.8508^\circ$	and	$L_{10} = 335^\circ$ (Pisces 5)
$RA_{11} = 18.3508^\circ$		$L_{11} = 19.8886^\circ$ (Aries 20)
$RA_{12} = 59.8508^\circ$		$L_{12} = 61.9623^\circ$ (Gemini 2)
$RA_1 = 101.3508^\circ$		$L_1 = 100.4289^\circ$ (Cancer 10)
$RA_2 = 119.8508^\circ$		$L_2 = 117.7527^\circ$ (Cancer 28)
$RA_3 = 138.3508^\circ$		$L_3 = 135.8725^\circ$ (Leo 16)

The values for houses 4-9 are found by the rule of opposites—same degree, opposite sign. The 4th house, for instance, will be defined in this example by Virgo 5.

6.2 The Equatorial Method (Regiomontanus):

$$\cot L = \cos E \cot (RA_{10} + CP) - \frac{\sin E \tan P \sin CP}{\sin (RA_{10} + CP)},$$

where CP = the given circle of position

$$\begin{aligned} CP &= 30^\circ \quad \text{for} \quad 11\text{th} \\ &= 60^\circ \qquad\quad 12\text{th} \\ &= 120^\circ \qquad\quad 2\text{nd} \\ &= 150^\circ \qquad\quad 3\text{rd}. \end{aligned}$$

Example: to find the cusps of the houses when L_{10} is Pisces 5:

1. If L_{10} is 335°, RA_{10} will be 336.8508°

$$\begin{aligned} \text{therefore} \quad RA_{10} + CP &= 6.8508^\circ \quad \text{for} \quad 11\text{th} \\ &= 36.8508^\circ \qquad\quad 12\text{th} \\ &= 96.8508^\circ \qquad\quad 2\text{nd} \\ &= 126.8508^\circ \qquad\quad 3\text{rd}. \end{aligned}$$

2. Then, for the 11th house:

$$\cot L_{11} = \cos 23.525^\circ \cot 6.8508^\circ - \frac{\sin 23.525^\circ \tan 53^\circ \sin 30^\circ}{\sin 6.8508^\circ}$$

$$= 10.4698^\circ$$

= Aries 10.

3. Similarly,

$$\begin{aligned} 12 &= 65^\circ\ 22' = \text{Gemini } 5 \\ 1 &= 100^\circ\ 25' = \text{Cancer } 10 \\ 2 &= 119^\circ\ 47' = \text{Cancer } 30/\text{Leo } 0 \\ 3 &= 135^\circ\ 31' = \text{Leo } 16 \end{aligned}$$

7. To find a body's azmith (AZ) on rising (r) or setting (s).

Formulas: $\cos AZ_r = \frac{\sin D}{\cos P}$; $AZ_s = 360^\circ - AZ_r$.

Example: with the sun at Pisces 5:

$$\cos AZ_r = \frac{\sin -9.7115^\circ}{\cos 53^\circ}$$

$$= -0.2083$$

$$AZ_r = 106.2779^\circ$$

This distance is measured from north through east.

8. To find a body's altitude (AL) and azimuth (AZ).

Formulas:

(1) $\sin AL = \sin D \sin P + \cos D \cos P \cos HA$

where HA is the distance of the body from the meridian as measured along the celestial equator;

(2) $$\cos AZ = \frac{\sin D - \sin P \sin AL}{\cos P \cos AL}$$

Example: to find AL and AZ at 10 a.m., when sun at Pisces 5:

(1) if time = 10.00 hrs, HA will be 30°; then,

$$\sin AL = \sin -9.7115^\circ \sin 53^\circ + \cos -9.7115^\circ \cos 53^\circ \cos 30^\circ$$
$$= 0.3789$$
$$AL = 22.2716^\circ$$

(2) $$\cos AZ = \frac{\sin -9.7115^\circ - \sin 53^\circ \sin 22.2716^\circ}{\cos 53^\circ \cos 22.2716^\circ}$$

$$= -0.8464$$
$$AZ = 147.8208^\circ$$

9. To find the oblique ascensions/descensions of the Significator and the Promittor.

In a given horoscope the following values are known or may be found:

P latitude
E obliquity of the ecliptic
DH distance at the horizon between the equator and the ecliptic
DE distance along the ecliptic from the horizon to a given body.

Then

(1) $\sin Z = \dfrac{\cos P}{\sin E} \sin DH$, where $DH = 90 - AZ_r$ (Formula 7)

(2) $$\tan \frac{R}{2} = \frac{\cos \frac{Z - DH}{2}}{\cos \frac{Z + DH}{2}} \cot \frac{P + E + 90}{2}$$

(3) $\cos Y = (\cos DH \sin DE \cos R) - (\sin DH \cos DE)$

(4) $\sin K = \dfrac{\cos DH}{\sin Y} \sin R$

(5) $\cos S = (\cos K \cos E) - (\sin K \sin E \cos (DE - Z))$

(6) Finally, the complement of S may be substituted for P(hi) in Formula 3, whereupon Formula 5 will give the oblique ascension/

descension of the Significator. The *same* value is retained for finding the acension/descension of the Promittor.

Example: to find the oblique descensions of the moon (Significator) and of the 8th house (Promittor) in Burton's horoscope:

$$\begin{aligned} P &= 52^\circ\ 20' \text{ (given)} \\ E &= 23^\circ\ 31'\ 30'' \text{ (by calculation)} \\ DH &= 18^\circ\ 19' \text{ (by Formula 7)} \\ DE &= 13^\circ\ 29' \text{ (by inspection)} \end{aligned}$$

Then

(1) $$\frac{\cos 52.333^\circ}{\sin 23.525^\circ} \sin 18.316^\circ = 0.481 \ (\sin Z)$$

$$Z = 28.766^\circ$$

(2) $$\frac{\cos 5.222^\circ}{\cos 23.541^\circ} \cot 82.929^\circ = 0.134 \ (\tan R/2)$$

$$R = 15.346^\circ$$

(3) $$(\cos 18.316^\circ \sin 13.483^\circ \cos 15.346^\circ) - (\sin 18.316^\circ \cos 13.483^\circ) = -0.092 \ (\cos Y)$$

$$Y = 95.292^\circ$$

(4) $$\frac{\cos 18.316^\circ}{\sin 95.292^\circ} \sin 15.346^\circ = 0.252 \ (\sin K)$$

$$K = 14.615^\circ$$

(5) $$(\cos 14.615^\circ \cos 23.525^\circ) - (\sin 14.615^\circ \sin 23.525^\circ \cos -15.283^\circ) = 0.790 \ (\cos S)$$

$$S = 37.809^\circ.$$

Then, with the moon at 222.25°, $D = -15.567^\circ$ (Formula 2).

$$\cos HD = -\tan (90 - S) \tan D$$

$$HD = 68.958^\circ$$

$OD = RA - (90^\circ - HD)$, and $RA = 219.788^\circ$ (cf. Formula 2).

Therefore $OD = 198.747^\circ$.

Similarly with the 8th house at 250.833°, $D = -22.149^\circ$

$\cos HD = -\tan (90 - S) \tan D$, as before.

$HD = 58.358^\circ$ and $RA = 249.238^\circ$

$OD = 217.596^\circ$.

The difference (the 'arc of direction') is $18.849^\circ = 18^\circ\ 51'$.