The Frequency of Total and Annular Solar Eclipses for a Given Place

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A statistical approach is used for finding the mean frequency of a total and an annular eclipse of the Sun at a given place on the surface of the Earth.

INTRODUCTION

A classical question concerning solar eclipses is the following one: How often can a total or an annular eclipse of the Sun be expected at a given point on the Earth's surface? In their classical textbook *Astronomy,* Russell, Dugan and Stewart write[1]:

> "Solar eclipses that are *total* somewhere or other on the earth's surface are not very rare, averaging one for about every year and a half. But *at any given place* the case is very different. Since the track of a solar eclipse is a very narrow path over the earth's surface, averaging only 60 or 70 miles in width, we find that in the long run a total eclipse happens at any given station only once in about 360 years."

These authors, however, give no details about how this mean frequency of one total eclipse every 360 years has been found. The theoretical calculation is not easy, by reason of the large variations of the length and width of the path from one eclipse to another. Even for a given eclipse, this width can vary widely along the path. For the total eclipse of 1981 July 31, for instance, the width was 57 km at the beginning of the path, reached a maximum value of 108 km near the middle, and then decreased to 51 km towards the end.

COMPUTATION

In order to recompute the answer given by Russell, Dugan and Stewart, and to find the mean frequency for annular eclipses, too, we attacked the problem statistically. For every solar eclipse in the period AD 1700 to 2299, the local circumstances at 408 "standard points" on the Earth's surface were calculated. The following standard points have been chosen: the points at latitudes +80°, +70°, +60°, etc., to −80° on the 24 meridians of longitudes +180°, +165°, +150°, etc., to −165°.

The calculation, made on an HP-85 microcomputer, proceeded automatically. After calculating the Besselian elements of an eclipse, the machine examined for each of the 408 standard points whether or not there was a total or an annular eclipse there, after which the next eclipse was calculated, etc. A total or annular eclipse at a standard point was considered to be visible if, and only if, at the time of maximum eclipse, the geometric altitude of the centre of the Sun's disk was positive.

A part of the computer outprint looks as follows:

1973 JUN 30	+45	+10	TOTAL
1973 JUN 30	+15	+20	TOTAL
1973 JUN 30	+0	+20	TOTAL
1973 JUN 30	−60	−10	TOTAL
1973 DEC 24	+90	+10	ANNULAR
1973 DEC 24	+60	+0	ANNULAR
1973 DEC 24	+30	+10	ANNULAR
1973 DEC 24	+15	+20	ANNULAR
1976 APR 29	+30	+10	ANNULAR
1976 APR 29	−45	+40	ANNULAR
1976 OCT 23	−75	−20	TOTAL
1977 APR 18	−15	−20	ANNULAR
1979 FEB 26	+60	+70	TOTAL
1979 AUG 22	+135	−60	ANNULAR
1979 AUG 22	+120	−60	ANNULAR
1979 AUG 22	+105	−60	ANNULAR
1979 AUG 22	+105	−70	ANNULAR
1979 AUG 22	+90	−70	ANNULAR
1980 AUG 10	+60	−20	ANNULAR
1981 FEB 4	−135	−40	ANNULAR
1983 JUN 11	−90	−20	TOTAL
1983 JUN 11	−105	−10	TOTAL
1983 JUN 11	−150	−10	TOTAL

where the second column gives the longitude, and the third column the latitude.

As mentioned above, the investigation was made over a time-period of six centuries, this value having been chosen in order to avoid any possible effect with a period of six centuries. It is well known, for instance[2], that the mean frequency of total *lunar* eclipses varies with time, the period being 586 years.

For the period 1700–2299, we found 665 total and 1208 annular eclipses at standard points, distributed over the various latitudes as indicated in Table I, columns (2) and (3). These values are plotted in the figure—see the two solid lines.

There is an evident *latitude effect* in the distribution of these total and annular eclipses. For instance, we see at once that total eclipses are less frequent in the zone of southern latitudes 30° to 80° than in the northern hemisphere. Annular eclipses are more frequent at latitudes 50°S to 80°S than near the equator.

These distributions are explained by the combination of the following effects:

(i) in the equatorial regions, total eclipses are more frequent, and annular eclipses are less frequent, than at higher latitudes because the equatorial regions are generally closer to the Moon, whose disk thus appears larger;

(ii) on the other hand, the lunar shadow moves in approximately the same direction as the rotating surface of the Earth. Therefore, the mean duration of solar eclipses at a given place will be longer, and their mean frequency less, than it would be for a non-rotating Earth. This is connected with the fact that the probability for the Sun to be eclipsed (expressed, for instance, in minutes per century) remains independent of the speed of rotation of the Earth. As a consequence, near the equator a solar eclipse will be somewhat rarer;

(iii) in the summer months, the Sun is a longer time above the horizon, increasing the frequency of visible eclipses there. In the northern hemisphere, this occurs around the time when the Earth is near the aphelion of its orbit, resulting in a smaller-than-average solar disk, thus favouring the occurrence of a total eclipse, and disfavouring that of an annular one; the opposite holds for the southern hemisphere.

As a consequence, the mean frequency of a total eclipse at a given point is higher in the northern hemisphere than in the southern one. That of an annular eclipse is higher in the northern hemisphere than in the equatorial regions, and still higher in the southern hemisphere.

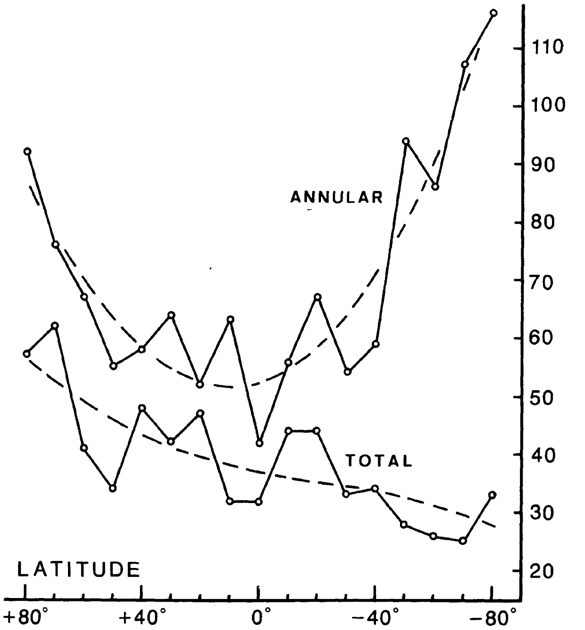

Figure 1. Number of total and annular eclipses at the standard points, for the period AD 1700 to 2299.

Using the method of least squares, we fitted a polynomial of the third degree to our data—see the dashed curves in the figure. This led us to the mean frequencies given in Table I, columns (4) and (5). For instance, at latitude 50° north, one can expect one total solar eclipse every 315 years, and one annular eclipse every 226 years.

In order to obtain the mean frequencies at a given point on the Earth's surface *generally,* we must take into account the variation of the circumference of a parallel of latitude. Having chosen the same number of standard points, namely 24, on each parallel of latitude, these points are closer to each other at high latitudes than near the equator. This fact cannot be neglected due to the latitude effect discussed above. As a consequence, the number of points and the number of events at each latitude ϕ should be weighted by $\cos \phi$ before the general mean can be deduced—see the Appendix.

We then, finally, obtain the following mean frequencies for any given point at the Earth's surface:

a total eclipse once in 375 years,

an annular eclipse once in 224 years,

which, by combination, gives an annular *or* a total eclipse every 140 years for a place chosen at random.

Because our results are based on a sample of observable eclipses, they are subject to uncertainty of about 16 years' standard error for total eclipses, 7 years for annular, and 4 years for both. Further error may arise because of the limited number of

Table I

Mean Frequencies of Eclipses

Latitude	*Number of eclipses at standard points*		*Mean time interval (years)*	
	Total	*Annular*	*Total*	*Annular*
(1)	(2)	(3)	(4)	(5)
+80°	57	92	254	166
+70	62	76	275	185
+60	41	67	295	205
+50	34	55	315	226
+40	48	58	333	246
+30	42	64	350	262
+20	47	52	364	274
+10	32	63	377	279
0	32	42	388	275
−10	44	56	398	264
−20	44	67	407	247
−30	33	54	417	226
−40	34	59	427	203
−50	28	94	441	180
−60	26	86	458	159
−70	25	107	481	140
−80	36	116	513	122

points on the Earth's surface used in the analysis. However, if the 600-year interval examined and the points considered are representative, total eclipses of the Sun visible at an average point on the Earth's surface may be a little less frequent than stated by Russell, Dugan and Stewart.

It should be noted that the obtained values are the *mean* frequencies of total and annular eclipses for a given place. Actually, these events take place at very irregular intervals for a given place. For instance, the eclipse of 2142 May 25 will be the first total solar eclipse at Antwerp, Belgium, for at least seven centuries. But only nine years later, on 2151 June 14, there will be another total one for the same city.

ACKNOWLEDGEMENT

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REFERENCES

1 Russell, H. N., Dugan, R. S. and Stewart, J. Q., *Astronomy*, I, 227, Boston, 1926.
2 Meeus, J., *J. R. astron. Soc. Canada*, **74**, 291 (1980).

APPENDIX

The calculation of the general mean frequency of eclipses

Let $F(\Omega)$ be the mean frequency of eclipse of a certain kind for the point $\Omega\,(\lambda, \phi)$ on the Earth. Then, the mean frequency for any given station on the Earth—thus averaged over all possible positions of such stations—is given by

$$\bar{F} = \frac{\int F(\Omega)\, d\Omega}{\int d\Omega}$$

where $d\Omega = \cos\phi\, d\phi\, d\lambda$. Since the frequency is independent of the longitude λ, we have $F(\lambda, \phi) = F(\phi)$, and the formula above reduces to

$$\bar{F} = \frac{\int_{-90}^{+90} F(\phi) \cos\phi\, d\phi}{\int_{-90}^{+90} \cos\phi\, d\phi}\,.$$

In our case, $F(\phi)$ is given for a limited number of latitudes $\phi_1 \ldots \phi_n$. Thus, the last formula can be approximated by

$$\bar{F} \approx \frac{\Sigma F(\phi_i) \cos\phi_i}{\Sigma \cos\phi_i}$$

since the chosen latitudes are uniformly distributed over the Earth's surface (+80°, +70°, etc. to −80°). The last formula leads to the procedure of averaging as indicated in the text.