Ulrich Voigt

Beda Venerabilis

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and the Great Easter Cycle

LIKANAS

Cover: George McFarlane

Preface

The 532-year cycle of the Easter date is a fundamental fact, and its representation as the product of the 19-year cycle of the Moon and the 28-year-cycle of the Sun in Bede's *De Temporum Ratione* is well-known and generally accepted. Nevertheless, this cycle is not quite as simple as is generally believed. In my paper I will argue that Bede's view is neither flawless, nor sufficient.

My main point is that the decomposition of the number 532 into a product must be supplemented by a decomposition into a sum.

My position is based on two arguments:

– Compound computistical cycles cannot be properly understood by a multiplicative analysis.

– The cyclus magnus decemnovenalis (the cycle of the Easter full moon and its weekday) and the cyclus magnus paschalis (the cycle of the Easter date) cannot be properly distinguished by a multiplicative analysis.

It took me quite a time to accept the fact that the Venerable Bede did not live up to the intricacies of the 532-year cycle. But considering that he did not discover it, and did not take into consideration the 95-year period which is an essential part of it, and after all was quite content with knowing its overall length of 532 years, the question if the Venerable Bede did understand the 532-year cycle, turns out to be a rhetorical question that demands the answer "No, not really."

For a more elaborate representation of the computus of additive structures, see Ulrich Voigt, *Zyklen und Perioden. Grundlagen der Komputistik*.

For its historical implications, see Ulrich Voigt, *Paul of Middelburg and the 437-year Period*, and *Die 418-jährige Periode in der PRAEFATIO SANCTI CYRILLI*.

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1. The common opinion

The first author to describe the 532-year Easter cycle that combines the moon with the date and the weekday, as the product of the 28-year cycle of the Sun and the 19-year cycle of the Moon, was the Venerable Bede. In *De Temporum Ratione*, Caput LXV we read:

> Circulus paschae magnus est, qui, multiplicato per invicem solari ac lunari cyclo, DXXXII conficitur annis.
>
> =
>
> The great Paschal cycle is (nothing else than) the product of the solar and the lunar cycle, thus comprising 532 years.

The same words are still used by scholars today, like e.g. Joachim Wiesenbach (1986):

> Die Multiplikation des 19 jährigen mit dem 28 jährigen sogenannten Sonnenzyklus ergibt den 532 jährigen großen Zyklus (annus magnus), das Nonplusultra der mittelalterlichen Komputistik. [1]

Time and again, Bede has been eulogized for his appropriate and concise description of the 532-year cycle, e.g. by Faith Wallis (1999): [2]

> Bede begins by clarifying the relationship between the great Paschal cycle of 532 years (the product of the 19 years of the lunar cycle and the 28 years of the cycle of weekdays) and the year of the Incarnation.

Faith Wallis here adopts Bede's presentation of the 532-year cycle as a product without change or commentary.
From equating the 532-year cycle with the product of the 19-year cycle of the Moon and the 28-year cycle of the Sun, the opinion arises that it cannot be properly understood otherwise.
Faith Wallis (1999) presupposes this belief when she writes: [3]

> Bede was the first computistical writer to expound the cyclical character of the Alexandrian reckoning correctly.

[1] Joachim Wiesenbach (ed.), *Sigebert von Gembloux Liber Decennalis*, Weimar 1986, Einleitung S. 49.

[2] Faith Wallis, *Bede The Reckoning of Time. Translated, with introduction, notes and commentary*, Liverpool 2004 (1999), 336 f.

[3] Loc. cit. 352.

At a stroke, the Great Cycle solved the problem of *computus* forever.

Bede, having understood the logic of the 532-year cycle so very well, is here credited to be the father of computus in the proper sense.
This view of the matter implies that before Bede (and, perhaps, his Irish and Anglo-Saxon predecessors) computus still had not reached the status of a science.

It is an almost inevitable consequence of this opinion that proper understanding of the 532-year cycle must be denied for the mathematician Victorius of Aquitaine who three centuries before the Venerable Bede composed a complete 532-year Easter table for the Roman Church and the Latin West.
From Victorius we have a (albeit very short) commentary on the nature of his 532-year Easter table written for the Archdeacon Hilarus who had asked him to undertake that work and who was to become Pope himself in AD 461.
Victorius says that he will carefully compile the facts concerning the Moon and the Sun (lunis atque temporibus) over 532 years, beginning with the year of the Passion, and that from these 532 years, 430 have already passed while 102 are still in the future. He then calls

532 years = 430 years + 102 years

a sum (ut quingentis triginta duobus annis omnis summa consistat), and proceeds as follows: [4]

> Quae summa ita cunctarum, quibus exempta est, seriem regularum sua revolutione conplectitur, ut eodem tramite et in id, unde est orta, revocetur et ad finem pristinum denuo circumacta perveniat.

=

> This sum thus comprises in its revolution the whole series of relevant data (regulares) from which it has been extracted, so that it repeats itself on the same route and will end at the same place from which it originally started.

[4] *Prologus Victorii Aquitani ad Hilarum archidiaconum*, sect. 10, from Bruno Krusch, *Studien zur christlich-mittelalterlichen Chronologie. Die Entstehung unserer heutigen Zeitrechnung*, Berlin 1938, 25.

Indeed, Victorius does not make mention of the 28-year cycle of the Sun, nor of the 19-year cycle of the Moon, but only refers to the corresponding data as if they came year by year, ending in a rather amorphous sum.

The first scholar to conclude that Victorius did not properly understand the cyclical nature of his own 532-year Easter table, was Bartholomew MacCarthy (1901): [5]

> This [Victorius, *Prologus* 10], it is hardly necessary to observe, reveals no acquaintance with 4 x 7; otherwise, the 28 would not have failed to be contrasted with the 19. Whence it follows that this great Cycle of 532 years was not derived by an Eastern source. The author, namely, worked by quadrennial period, and found the solar and lunar date recurring after the 133rd.The Victorian period was accordingly 133 x 4. To call it *Great Paschal* in the sense that it was consciously based on the formula 28 x 19, or 28 x 4 x 7, is a complete misnomer.

In fact, if the 532-year cycle is defined by the equation 532 = 28 x 19 (or 532 = 19 x 4 x 7), it cannot be properly understood by the equation

$$532 = 133 \times 4.$$

Following *De Temporum Ratione* Cap. LXV (cited above), MacCarthy thus excludes sufficient understanding of the cycle for Victorius by definition.

Here are some striking examples that show to what extent MacCarthy's conclusion has become commonplace among scholars:

Charles W. Jones (1943): [6]

> [Victorius] Having written a 532-table by accident […].

Faith Wallis (1999) : [7]

> Victorius noticed that his data began to repeat after 532 years, but did not know why.

[5] Bartholomew MacCarthy, *The Annals of Ulster*, vol. IV, Dublin 1901, lxxxv.

[6] Charles W. Jones, Introduction, in: Jones (ed.), *Beda Venerabilis, De temporum ratione*, Cambridge 1943, 64.

[7] Faith Wallis, loc. cit., 352.

Leofranc Holford-Strevens (1999): [8]

> The 19- and 28-year-cycles combine to form a 532-year-sequence […] stumbled into […] by Victorius of Aquitaine […].

George Declerq (2002): [9]

> The only possible conclusion is therefore, as C. W. Jones has stated, that during the establishment of his new table Victorius found "by accident" that at a certain point the Easter dates began repeating themselves.

In summa: Common opinion has it that from Victorius of Aquitaine to the Venerable Bede considerable progress has been made concerning the fundamental concept of computistical cycles, because the multiplicative structure of the 532-year Easter cycle was only then discovered. When Wiesenbach (1986) referred to the 532-year cycle as the ultimate of medieval computus, he wanted to say en passant that this cycle did not yet belong to pre-medieval times and that presenting it in the way *De Temporum Ratione* does, was a remarkable historical achievement not easily overrated.

Having now understood the common opinion about the 532-year cycle that combines the Moon, the Sun and the Weekday, we can proceed to discuss its presuppositions.

2. Questioning the common opinion

Confusion of attribute and essence.

Bede did not simply say that the said decomposition into a product is an attribute of the 532-year Easter cycle, an essential property by which it can be established, but he equated the two, i.e. he defined the 532-year cycle by its decomposition into a product of the 19-year cycle of the Moon and the 28-year cycle of the Sun, implying that the essence of the 532-year cycle is its decomposition into that product, and that it cannot be understood otherwise. We have seen modern

[8] Bonnie Blackburn & Leofranc Holford-Strevens, *The Oxford Companion to the Year. An exploration of calendar customs and time-reckoning*, Oxford 1999, 802.

[9] George Declerq, "Dionysius Exiguus and the Introduction of the Christian Era", in: *sacris erudiri* 41 (2002), 183 f.

scholars following him in this point. Now, I am of opinion that in doing so they fall into a trap.
Just consider “Four is the square of two.” If it is well understood, there is nothing wrong with this statement, as indeed 4 = 2 x 2. But if taken as a definition of the essence of the number four, we might be induced to consider propositions like “Four is the successor of three” or “Four is a multiple of one” as insufficient and even inappropriate, and conclude that those who describe the number four in these odd ways, cannot have properly understood its essence.
Let us beware of this trap, and in order to make it more acceptable, let us reduce Bede’s position to the following statement:

> *The decomposition of the 532-year period into the product of the 19-year cycle of the Moon and the 28-year cycle of the Sun proves without further argument that 532 years is the cycle of the combination Easter full moon / weekday.*

This cycle of 532 years is commonly called the cyclus magnus decemnovenalis, and it is so to speak the mother of the cyclus magnus paschalis, the cycle of the Easter date that has the same length.

The 532 year cycle is not the product of cycles.

Though it sounds so very natural, 'product of cycles' is actually rather misleading, and for two reasons.
Firstly, as the cycles are measured in years, their product is measured in square years, an unwieldy consequence in view of the fact that the product is supposed to equal a cycle that is measured in years.
I think what is really meant when we say 'product of cycles' is 'multiple of a cycle' rather, just like is the case with 3 units of 4 kg potatoes each, as this can properly be described by

12 kg potatoes = 3 x (4 kg potatoes),

as opposed to the senseless equation

12 kg potatoes = (3 kg potatoes) x (4 kg potatoes)

which implies 1 kg = 1 kg^2.
Secondly, 'product' seems to imply commutativity, like Bede’s

multiplicato per invicem solari ac lunari cyclo that does not exclude changing the order of the two cycles involved, but clearly includes the idea of

532 years = 28 x (19 years),

as well as

532 years = 19 x (28 years).

But contrary to

3 x (4 kg potatoes) = 4 x (3 kg potatoes),

we cannot say

19 x (28 years) = 28 x (19 years),

without running into difficulties because only one of the two terms makes sense.

The reason is obvious. While '28 years' denotes a definite number of days in the Julian calendar, '19 years' does not. But 'multiple of a quantity' only makes sense if the quantity is a definite quantity.

'Multiples of years': In the Julian calendar, only multiples of 4 years (= 1,461 days) constitute definite numbers of days.

On the other hand, 19 years are equal to 6,940 days if they begin with a leap year, and equal to 6,939 days if they begin with a common year, which makes '19 years' an ambiguous quantity.

What is meant when we say 532 years = 28 x (19 years), is in fact a sum of 28 summands of 19 years each that are of unequal length.

> *I thus come to the conclusion, that the* cyclus magnus decemnovenalis *is a multiple of the 28-year cycle of the Sun as well as of the 4-year cycle of the leap year.*

With that, we can now make out more clearly the objection raised by Bartholomew MacCarthy against Victorius' presentation of the cyclus magnus decemnovenalis. MacCarthy thought that this cycle can only be demonstrated by its decomposition into a multiple of the 28-year cycle of the Sun, and he doubted that this was equally possible by its decomposition into a multiple of the 4-year cycle of the leap year.

Bede's famous 532-year Easter table is a sequence of twenty-eight 19-year tables, which suggests 532 years = 28 x (19 years), but hardly 532 years = 19 x (28 years).

The 532-year cycle cannot be properly understood as a multiple.

Thus the wide-spread opinion that the cyclus magnus decemnovenalis has to be understood as the product of the 19-year cycle and the 28-year cycle is patently false, and even without sense, and we may try to replace it by

> *The* cyclus magnus decemnovenalis *must be understood as a multiple of the 28-year cycle or of the 4-year cycle.*

But even in this form, the opinion cannot be defended because the structural difference between the cyclus magnus decemnovenalis and the cyclus magnus paschalis cannot be explained in this way. Nor do I want to imply that Bede had the multiple of a cycle in mind when he spoke about a product of cycles.

Fact remains, that Bede, by confining his analysis to the multiplicative structure, was unable to distinguish properly between these two cycles of equal length, and the same is true for his medieval successors:

> *Compound computistical cycles like the 28-year cycle of the Sun, the 532-year cyclus magnus devemnovenalis and the 532-year cyclus magnus paschalis must be understood as sums of parts of different length rather than as multiples of a constant length.*

To make this clear, we have to go into more detail.

3. Cyclus solaris

In *De Temporum Ratione*, Caput LIII we read

… epactae solis, id est, concurrentes septimanae dies unius semper ternos per annos, duorum autem per annum bissextilem …usque ad septimum numerum adiectione crescentes, quarum circulus habet annos quater septenos, id est, XXVIII, quia nimirum non ante potest consummari quam bissextus, qui quarto redire solet anno, cunctos septimanae dies contingat, dominicam videlicet, sextam feriam, quartam feriam, secundam feriam, Sabbatum, quintam feriam, tertiam feriam, hoc etenim illos ordine percurrit.

=

… the epacts of the Sun, i.e. the concurrentes of the Week, grow over three years from one year to the next for one day, and at the leap year for two, until the number 7 is completed; their cycle comprises 4 x 7, that is 28 years because without any doubt it cannot come to its end until the leap day that recurs every fourth year has run through a whole week, namely Sunday, Friday, Wednesday, Monday, Saturday, Thursday, Tuesday, in that order.

When Bede wrote this text, he had before his eyes something like the following table of the concurrentes over a period of 28 years: [10]

B	**C**	**C**	**C**
1	2	3	4
6	7	1	2
4	5	6	7
2	3	4	5
7	1	2	3
5	6	7	1
3	4	5	6

The concurrens of a year j is the weekday of March 24 of that year:

$$\text{con}\ (j) = W\ (\text{March } 24\ j).$$

The table begins with a leap year and con = 1, e.g. with AD 720.
Bede refers to the first column of our table as the sequence of weekdays of the leap day, which implies that contrary to Roman usage he considered the bissextum not as the day following February 23, but as the day before February 25 because only then its weekday matches with con.
The table shows more clearly than words could describe it, what happens with the weekday of a date from one year to the next. There are seven groups of four concurrentes each. They are separated by +2 instead of +1 because of the leap day. Only after 28 (= 7 x 4) years, the sequence repeats itself. It is obvious that Bede's argument is flawless.

[10] B = annus bissextus, C = annus communis, 1 = Sunday, 2 = Monday, etc.
The sequence 1 – 6 – 4 – 2 – 7 – 5 – 5 – 3 finds no comment in Faith Wallis' commentary.

Thus the cyclus solaris, i.e. the cycle of the concurrentes, is established. Only after 28 years does a date of the Julian calendar recur on the same weekday and on the same place of the leap year cycle. This means to say that only after 28 years, all dates recur on the same weekday collectively.
On the other hand, the question after how many years a date will recur on the same weekday (regardless of the place in the leap year cycle), does not admit of a simple answer because it depends on the place of the year in the leap year cycle.
This place I number from $\nu = 0$ for the leap year to $\nu = 3$ for the first year before the leap year, i.e. by $\nu = j \bmod 4$ for the years j AD.
Each ν has a definite sequence σ_ν for the recurrence of the combination date / weekday, as follows:

$\sigma_{\nu=0}$	=	6	11	6	5
$\sigma_{\nu=1}$	=	6	5	6	11
$\sigma_{\nu=2}$	=	11	6	5	6
$\sigma_{\nu=3}$	=	5	6	11	6

These four sequences are so many permutations of the same 11 6 5 6 – pattern, and are found by inspection from the above table. They apply without exception if we presuppose that the calendaric year begins with the leap day (as Bede seems to suggest), or with March 1 (as is commonly done by the computists).
σ is a sum of four parts that add up to 28, so we have an additive decomposition of the 28-year period that proves this period to be a cycle of the combination date / weekday. Please note that 28 (= 4 x 7) arises independently of the fact that 4 and 7 are relatively prime.
σ is cyclical and symmetric, and can be written in a circle

$$\begin{array}{ccc} & 6 & \\ 11 & & 5 \\ & 6 & \end{array}$$

which I imagine as a clock-face that is easily memorized, and indeed has to be read clockwise, while ν moves crosswise like South – North / West – East as follows:

1

2 3

0

It goes without saying that σ proves that the weekday recurs on the date and on the same place of the leap year cycle only after 28 (= 11 + 6 + 5 + 6) years.

Example: 1230 August 10 Saturday. What is the next year with the same combination date / weekday? What are all the years that satisfy this condition?
From ν (1230) = 2 it follows that $\sigma_{j = 1230} = \sigma_{\nu=2}$ = 11 6 5 6. Hence, the next following year with August 10 Saturday is 1241 (= 1230 + 11), and the corresponding years for August 10 Saturday are:

··· 1202 1213 1219 1224 **1230** 1241 1247 1252 1258 ···

Obviously, the decomposition of 28 into a sum (28 = 6 + 11 + 6 + 5) renders a result that is at once more exact and more useful than its decomposition into a product (28 = 7 x 4).

De Temporum Ratione Cap LIII shows that the Venerable Bede had no difficulty to answer these questions and find the corresponding years to a given combination date / weekday: [11]

> Cuius circuli talis est cursus, ut quaecunque bissextili anno sunt concurrentes, ipsae et ante quinquennium fuerint, et post annos VI futurae sint. Quae primo post bissextum anno sunt, eaedem et ante annos XI transierint, et post VI redeant. Quae secundo post bissextum, eaedem et ante annos VI transierint, et post XI remeent. Quae tertio post bissextum, ipsae et ante VI annos praeterierint, et post V revertantur. Et huius ordo discretionis cunctos annorum vertentium complectitur dies.
> =
> This cycle [the 28-year cyclus colaris] runs as follows: the concurrentes of the leap years are the same five years before and six years after. The concurrentes of the first years after the leap year are the same eleven years before and six years after. The concurrentes of the second years after the leap year are the same six years before and eleven years after. The concurrentes of the third years after the leap

[11] This passage finds no comment in Faith Wallis' commentary.

year are the same six years before and five years after. This pattern applies to all the days of the revolving year.

In short, the recurrence of the weekday corresponds to the following table:

ν	0	1	2	3
before / after	5 / 6	11 / 6	6 / 11	6 / 5

This, of course, is the 11 6 5 6 – pattern, arranged as a double sequence which for easy memorisation can be represented as a small bridge:

		11	6		6	11		
5	6						6	5

The bridge has two steps up, and two steps down, and has to be passed from left to right. It begins and ends with 5, and the symmetry is obvious. Here, ν moves parallel:

	ν = 1	ν = 2	
ν = 0			ν = 3

Example: 1230 August 10 Saturday.
From ν = 2 (= 1230 mod 4) we find ourselves on top of the bridge on the right side (6 / 11 = before / after), thence we know that the combination August 10 / Saturday recurred in 1224 and in 1241.
Considered as a mnemonic, Bede's bridge is somewhat less convenient than our clock-face, but it is, of course, computistically equivalent.
Bede wrote Cuius circuli talis est cursus, i.e. he made a distinction between the cycle and its course. He did not realize that there is no difference, and that the pattern he described was identical with the cycle. I believe that this happened because at every step he laboriously looked forward and backward instead of only forward or backward.
But this is to say that Bede did not fully understand the structure of the 28-year cycle that combines the date with its weekday, which is not multiplicative but additive.

If he had fully understood the significance of the sum

$$28 = 6 + 11 + 6 + 5,$$

he would probably have applied it to the 532-year cycle. But he did not, nor did any of his successors during the Middle Ages.

4. Cyclus magnus decemnovenalis

This cycle combines the 19-year cycle of the moon with the 4-year cycle of the leap year and the 7-day cycle of the weekday, and is a cycle of 532 (=19 x 4 x 7) years: Only after 532 years does the Easter full moon recur on the same weekday and on the same place of the leap year cycle. Only after 532 years, all Easter full moons recur on the same weekday collectively.

Now, the positon of the year in the leap year cycle is of little interest, whereas the combination Easter full moon / weekday, implying the Easter date, is crucial.

But the question after how many years an Easter full moon will recur on the same weekday (regardless of the position in the leap year cycle), does not admit of a simple answer because it depends on the place of the year in the leap year cycle.

Again, we have an additive structure, and definite sequences $\sigma_{\mathbf{v}}$: [12]

$$\sigma_{v=0} = 247 \quad 95 \quad 95 \quad 95$$

$$\sigma_{v=1} = 95 \quad 247 \quad 95 \quad 95$$

$$\sigma_{v=2} = 95 \quad 95 \quad 247 \quad 95$$

$$\sigma_{v=3} = 95 \quad 95 \quad 95 \quad 247$$

These four sequences all belong to the same pattern, and follow easily from applying the cyclus solaris ("six–five–six–eleven") to the multiples of 19.

It goes without saying that σ proves that the Easter full moon recurs on the same weekday and on the same place of the leap year cycle after 532 (= 247 + 95 + 95 + 95) years.

[12] I use the same letter σ as above. I trust that the context will make the matter clear enough so that confusion will not arise.

σ is cyclical and symmetric, and can be written in a circle

$$\begin{array}{ccc} & 95 & \\ 247 & & 95 \\ & 95 & \end{array}$$

to be combined clockwise with ν, as follows:

$$\begin{array}{ccc} & 3 & \\ 0 & & 2 \\ & 1 & \end{array}$$

Example: Easter full moon 1136 on March 21 Saturday. What is the next year with the same combination Easter full moon / weekday? What are all the years that satisfy this condition?
Now, from ν (1136) = 0 it follows that $\sigma_{j=1136} = \sigma_{\nu=0}$ = 247 95 95 95. Hence, the next following year with the Easter full moon on March 21 Saturday is 1383 (= 1136 + 247), and the corresponding years are:

… 604 851 946 1041 **1136** 1383 1478 1573 1668 …

Obviously, the decomposition of 532 into a sum (532 = 247 + 95 + 95 + 95) renders a result that is at once more exact and more useful than its decomposition into a product (532 = 19 x 28).

Let us now cast a glance on the overall representation of the 532-year cycle. It is natural to conceive a matrix of 28 rows and 19 columns (or vice versa) so that there are 532 places for 532 years.
Indeed, so easily this matrix suggests itself, that scholars tend to confound its 532 places with the 532 years that are counted by them. When Bede wrote that the 532-year cycle is the product of the 28-year cycle and the 19-year cycle, he probably did not think about years but of a matrix of places and the product of 28 rows and 19 columns.
In a 11th century computistical tractate,[13] the 532-year cycle of the Easter full moon and her weekday, i.e. the 532-year cycle that combines epacts and concurrents, is deduced in this manner:

[13] Alfred Lohr, *Der Computus Gerlandi. Edition, Übersetzung und Erläuterungen*, Stuttgart 2013, liber primus, I (expositio tabulae).

Superioris igitur paginae ratio haec est: Habet namque numerum epactarum in latitudine, numerum vero concurrentium in longitudine. Multiplicati igitur vel latitudine per longitudinem vel longitudine per latitudinem tota summa excrescit in DCCCII.

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The table on the previous page has to be understood as follows: The columns are counted according to the [nineteen] epacts, the rows according to the [twenty-eight] concurrents. The product of the [number of] rows with the [number of] columns or vice versa is 532.

Here, the numbers – the epacts and concurrents –are non-dimensional, and the ensuing 532-year cycle clearly is the product of the 28-row cycle of the epacts and the 19-column cycle of the concurrents.

But what information can be drawn from such a matrix except that these cycles combine to a 532-year cycle?
If we consider a 7 x 4 – matrix by combining the week and the leap year cycle, we get 28 places with 19 years each that share the same Easter full moon and the same place in the leap year cycle.
But the place in the leap year cycle is of little interest, being no part of the definition of the Easter date, and this is to say that the idea of a 28 x 19 – matrix is perhaps not the best arrangement of 532 places with regard to comprehensibility.
Otto Neugebauer (1979)[14] found the solution. He proposed a 7 x 19 – matrix, combining the week and the Easter full moon, and got 133 places with four years each that share the same combination Easter full moon / weekday. Of course, these four years are related by σ_v as discussed above. This is relevant, and we have discussed it already. Neugebauer’s matrix reveals the additive structure of the 532-year cycle of the Easter full moon and her weekday.

This said, we can now turn to Bede’s *De Temporum Ratione* and its presentation of the 532-year cycle.
To make clear that the cyclus magnus decemnovenalis is a 532-year cycle, Bede argued just in the same way as he did in respect to the 28-year cyclus solaris:

[14] Otto Neugebauer, *Ethiopic Astronomy and Computus*, Wien 1979, 223 f.

> Notandum sane quod huius gyri solaris, qui XXVIII annis peragitur, causa facit decennovales circulos XXVIII debere compleri, priusquam idem per omnia paschalis observantiae cursus in seipsum redeat, ut omnis nimirum huius circuli annus caput circuli decennovenalis instituat. Itemque annus quisque circuli decennovenalis huius caput adsequatur, ac per hoc tota paschalis observantiae series non minus quingentis triginta duobus annis explicetur.

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> It should be noted that because of this 28-year cycle of the Sun, twenty-eight 19-year cycles have to be full before all data that are of relevance for the Easter date repeat themselves. Thus every year of this cycle will be the first year of the 19-year cycle, and every year of the 19-year cycle will be the first year of this cycle. In this way, the whole series of Easter data will not be complete in less than 532 years.

Again, Bede's reasoning is flawless, but contrary to his reasoning in respect to the 28-year cycle he not only misses the additive structure of the cycle but also the means for understanding it.

If Bede had considered the question of finding the years to a given combination Easter full moon / weekday, he certainly would have found this table:

ν	0	1	2	3
before / after	95 / 247	95 / 95	95 / 95	95 / 247

giving rise to a nice bridge as follows

	95	95	95	95	
95	247			95	247

with two steps up, and two steps down, to be passed from left to right. Again, ν moves parallel:

	ν = 1	ν = 2	
ν = 0			ν = 3

If Bede had understood this pattern, he certainly would have inserted a chapter on the 95-year period in *De Temporum Ratione*. But he did not. Though Dionysius Exiguus bequeathed a 95-year Easter table to the Latin world, Bede did not try to understand the 95-year period, in fact he did not lose a word about it.

On the other hand, Werner Bergmann (2010)[15] brought to light that the Irish monk Dicuil who in 815 wrote a *Liber de Astronomiae* at the Carolingian court for Louis the Pious did know the 247 95 95 95 – pattern. But, alas, Dicuil did not work out the consequences and, as he wrote in a somewhat enigmatic style, his book had no effect on medieval computus.

The 247 95 95 95 – pattern might also have been known by Gerlandus in the 11th century, who wrote that the combination epact 12 / concurrens 5 (i.e. Easter full moon march 24 Thursday) of the year AD 12 recurs only in AD 259. But he showed no awareness of the correlation with the 532-year cycle. [16]

5. Cyclus paschalis

We have seen that Bede and his successors had no tool to calculate the years to a given combination Easter full moon / weekday, and much less, of course, to the Easter date. The standard German book of chronological reference, Hermann Grotefend's *Taschenbuch der Zeitrechnung* [17], gives the years to the Easter dates „old style" for AD 800 – 1700. The historian who for some reason or other needs an Easter date outside of this interval, is left alone.

The Easter date is by definition the Sunday after the Easter full moon, and it goes almost without saying that its cycle is a cycle of equal length with the cyclus magnus decemnovenalis. But when Bede wrote:

> Circulus paschae magnus est, qui, multiplicato per
> invicem solari ac lunari cyclo, DXXXII conficitur annis.

[15] Werner Bergmann, "Dicuils Osterfestalgorithmus im Liber de astronomia", in: Immo Warntjes & Dáibhí Ó Cróinín (eds.), *The Easter Controversy of Late Antiquity and the Early Middle Ages. Proceedings of the 2nd International Conference on the Science of Computus in Ireland an Europe, Galway, 18 – 20 July, 2008*, Brepols 2012, 242 – 287. Bergmann, who did not know the pattern beforehand, was much surprised with his find, and erroneously thought that it was a discovery of Dicuil. Ulrich Voigt, "Dicuil (um 815)", in *Zyklen und Perioden*, 130 ff.

[16] Alfred Lohr, *Der Computus Gerlandi. Edition, Übersetzung und Erläuterungen*, Stuttgart 2013, 152 f.

[17] Hermann Grotefend, *Taschenbuch der Zeitrechnung des deutschen Mittelalters und der Neuzeit*, Hannover 1898 (and many editions since).

he made no small mistake in his statement that the cycle of the Easter date is a deduction from the cycle of the Sun and the cycle of the Moon, because it is not. The definition “Sunday after …” adds something quite tricky to the cycle of the Easter full Moon and her weekday with the effect that a cycle arises of equal length, but my no means of equal structure.

This mistake of Bede has become a common one. Even Otto Neugebauer (1979), when he so cleverly unveiled the 247 95 95 95 – pattern in the 7 x 19 – matrix, did not stop to consider the Easter date, and spoke indiscriminately of “the 532-year table”.

As far as I know, the additive structure of the Easter cycle has never been expounded, though it is quite easy to find, as it follows empirically from the 532-year Easter table.

To make myself clear, I need a few definitions.

In place of Easter dates I use their difference ***easter*** *to March 21.*

There are 35 Easter dates from March 22 to April 25, and consequently 35 **Easter numbers** $1 \leq \text{easter} \leq 35$.

Example: 1136 Easter Sunday March 22 <=> easter (1136) = 1.

In place of the date of the Easter full moon I use its difference ***g*** *to March 21, called its* ***Grenzzahl****.* [18]

There are 28 Easter full moons from March 21 to April 19, and consequently 28 Grenzzahlen $0 \leq g \leq 28$.

Example: 1136 Easter full moon March 21 <=> g (1136) = 0.

The question what years belong to a given Easter date, gives rise to five different patterns, and not just to one only as was the case with the corresponding question of the Easter full moon. These patterns are so many additive structures. I denote them equally by the letter σ.

The Easter numbers thus fall into five **categories** according to these patterns.

The following table shows how the Easter numbers are related to the five categories.

[18] This definition is a common one, while "easter" is only implicitely used by scholars.

	easter	σ
(i)	01 35	247 95 95 95
(ii)	02 03 33 34	163 84 11 84 11 84 11 84
(iii)	04 31 32	79 84 11 73 11 11 73 11 11 73 11 84
(iv)	05 06 08 09 11 12 14 17 19 20 22 23 25 27 28 30	05 79 11 73 11 11 62 11 11 11 62 11 11 73 11 79
(v)	07 10 13 15 16 18 21 24 26 29	51 11 11 11 62 11 11 73 06 05 79 05 06 73 11 11 62 11 11 11

σ is cyclical and symmetric, and can be written in a circle as follows:

(i)

95

247 95

95

(ii)

84 11 84

163 11

84 11 84

(iii)

84 11 73 11 11

79 73

84 11 73 11 11

(iv)

79 11 73 11 11 62 11

05 11

79 11 73 11 11 62 11

(v)

	11	11	11	62	11	11	73	06	05	
51										79
	11	11	11	62	11	11	73	06	05	

The sequences are so many **Easter circles**, consisting of **parts**, and with two **vertex parts** each.
Of course, the parts always add up to 532. They are ultimately composed by the numbers 5, 6, and 11, i.e. by the elementary parts of the 28-year cycle of the Sun. The length of the sequences varies from four to twenty parts.
The parts p are numbers that represent intervals of p years in the Julian calendar which I call **p-periods**.

For the sake of simplicity, when talking about parts,
I will not distinguish between number and period.

p-periods are not arbitrary periods, as they connect years with identical Easter numbers. Thus, p = 247 as part of category (i) applies only to leap years which makes it a period of a definite length, and the same is true for p = 95 in the same group which only applies to common years. The same is also true for the vertex parts in the other groups.[19]
Every part of the Easter circles can be taken as their head. I chose one of the vertex parts. For (i – iv) there is only one such part, that does not appear twice in the circle. For (v) I chose the smaller one. This form I consider to be a canonical representative of the set of corresponding circles. The canonical circles are functions of the Easter numbers and of the categories, and we can write e.g.

$$\sigma_{\text{easter=03}} = \sigma_{(\text{ii})} = 163\ 84\ 11\ 84\ 11\ 84\ 11\ 84.$$

[19] Those vertex parts which I use in the above table as head parts, apply to years that satisfy v = 0 except for the head p = 05 of (iv) that applies to years that satisfy v = 3. Compare the table on p. 26.

In view of the symmetry, we can shorten the form to

$$\sigma_{(i)} = «247\ 95\ 95»$$

$$\sigma_{(ii)} = «163\ 84\ 11\ 84\ 11»$$

$$\sigma_{(iii)} = «79\ 84\ 11\ 73\ 11\ 11\ 73».$$

$$\sigma_{(iv)} = «05\ 79\ 11\ 73\ 11\ 11\ 62\ 11\ 11»$$

$$\sigma_{(v)} = «51\ 11\ 11\ 11\ 62\ 53\ 06\ 05\ 79».$$

The same symmetry applies to the cyclus solaris, and we can write

$$\sigma_{(sol)} = «\ 5\ 6\ 11»,$$

though there is no notable advantage in doing so.

Now, to apply these patterns to a given year, we must know its place in its circle.

Only in category (i) cyclus magnus decemnovenalis and cyclus magnus paschalis coincide, and this is the only case, where σ depends solely on ν:

$$\sigma_{(i)} = \sigma_{\nu}.$$

In words:

> *The* cyclus magnus decemnovenalis *is equivalent to the* cyclus magnus paschalis *reduced to* easter = 1 *and* easter = 35. [20]

This follows from

easter = 1 <=> Easter full moon March 21 (g = 0) / Saturday,

and

easter = 35 <=> Easter full moon April 18 (g = 28) / Sunday.

Example (i): 1014, Easter Sunday April 25.

$\nu\ (1014) = 2 \Rightarrow \sigma_{j=1014} = \sigma_{\nu=2} \Rightarrow$ the corresponding years are

··· 482 577 672 919 **1014** 1109 1204 1451 1546 ···

20 Knowledge of the 247 95 95 95 – pattern in respect of the Easter dates March 22 and April 25 is shown in the *Praefatio cycli paschali*, a Frankish tractate from the early 8th century. See Arno Borst (ed.), *Schriften zur Komputistik im Frankenreich von 721 bis 818*, Teil 1, Hannover 2006, 329–347. For a more detailed discussion, see U. Voigt, *Zyklen und Perioden*, p. 132 ff.

The categories (ii - v) are not that simple.
To every Easter number $1 \leq \text{easter} < 35$ the following table exhibits the first year $0 \leq A < 532$ to which its canonical sequence applies. We can call A = A (easter) the **Easter beginning** of the Easter number.

easter	A	easter	A	easter	A	easter	A	easter	A
01	72	08	403	15	492	22	95	29	184
02	4	09	335	16	424	23	27	30	363
03	384	10	468	17	71	24	160	31	216
04	316	11	115	18	204	25	339	32	64
05	243	12	47	19	383	26	472	33	444
06	91	13	180	20	315	27	119	34	292
07	224	14	359	21	448	28	51	35	140

Example (ii): 1269

easter (1269) = 3 =>

$\sigma_{j=1269} = \sigma_{(ii)} =$ «163 84 11 84 11» and A (easter=3) = 384.

Applying σ to A + 2 x 532 = 384 + 1064 = 1448,
we get easter = 3 for the years

··· 916 1079 1163 1174 1258 **1269** 1353 1364 **1448** 1611 ···

Example (iii): 1190

easter (1190) = 4 =>

$\sigma_{j=1190} = \sigma_{(iii)} =$ «79 84 11 73 11 11 73» and A (easter=4) = 316.

Applying σ to A + 2 x 532 = 316 + 1064 = 1380,
we get easter = 4 for the years

··· 1095 1106 1117 **1190** 1201 1212 1285 1296 **1380** 1459 ···

Example (iv): 1553

easter (1553) = 12 =>

$\sigma_{j=1553} = \sigma_{(iv)}$ = «05 79 11 73 11 11 62 11 11» and A (easter=12) = 47.

Applying σ to A + 3 x 532 = 47 + 1596 = 1643,
we get easter = 12 for the years

··· 1363 1374 1385 1396 1469 1480 **1553** 1564 **1643** 1648 ···

Example (v): 1286.

easter (1286) = 24 =>

$\sigma_{j=1286} = \sigma_{(v)}$ = «51 11 11 11 62 11 11 73 06 05 79» and
A (easter=24) = 160.

Applying σ to A + 2 x 532 = 160 + 1064 = 1224,
we get easter = 24 for the years

··· **1224** 1275 **1286** 1297 1308 1370 1381 1392 1465 1471 ···

A nice task for mnemonic construction and mental calculation:

Given an easter number $1 \leq$ easter ≤ 35, to know the easter beginning, and the category with its canonical sequence. To compute the years for the given easter number.

6. Dionysius Exiguus and Beda Venerabilis

The computus transmitted by *De Temporum Ratione* to the Middle Ages, is nothing other than the Alexandrian computus transmitted two centuries before by Dionysius Exiguus to the Latin West – except for one detail. While Dionysius composed a 95-year Easter table and did not even make a mention of the 532-year cycle, Bede, on the contrary, composed a 532-year Easter table and did not even make a mention of the 95-year period. The 95-year period and Easter table were not the invention of Dionysius, but belonged to Alexandrian computus and chronology, so we have a nice juxtaposition of late Antique computus Alexandrian style and Medieval computus Bedean style.

Ever since Denis Pétau, who first noted this disagreement between Dionysius and Bede,[21] scholars have tried to interpret it as evidence for the superiority of Bede over Dionysius, and of the Middle Ages over Late Antiquity. After all, the 532-year period is a perfect cycle for the Easter full moon and her weekday as well as for the Easter date, while the 95-year period is not.

Now we have seen that the 95-year period and the 532-year cycle cannot be understood independently from each other, as each of them presupposes the other. This means to say that the Alexandrians simply presupposed knowledge of the 532-year cycle. There is no reasonable explanation of them having found the 95-year period without taking account of the 532-year cycle.[22] On the other hand, it is quite easy to find the 532-year cycle without any insight into its structure, as you have only to multiply 4, 7, and 19, the three basic numbers of Alexandrian computus. Bede and his followers did not understand the structure of the 532-year cycle, else they would have talked about the 95-year period at length. Even worse, by restricting themselves to a multiplicative approach, they fundamentally went wrong because the cyclus solaris, the cyclus decemnovenalis, and the cyclus paschalis, are so many additively ordered objects, and cannot be duly understood by a multiplicative analysis.

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[21] *Opus de doctrina temporum*, Paris, 1627, tom. 2, lib. 12, cap. 3. See U. Voigt, *Zyklen und Perioden*, 13 ff.

[22] U. Voigt, *Zyklen und Perioden*, Kap. 6 ("Die Zahlen 95 und 532").

7. Additive decompositions of the number 532

The multiplicative decomposition of a number is, mathematically speaking, a clear-cut matter because basically there is one and only one such decomposition. An additive decomposition, on the other hand, is never that simple. In fact, it only becomes visible in the light of a definite question generating it.

The questions that generate the two additive decompositions characteristic for Alexandrian computus and chronology,

$$532 = 3 \times 95 + 247$$
$$532 = 95 + 437,$$

are "After how many years does the Easter full moon recur on the same weekday?" and "After how many years does the Easter full moon change in a minimal way?" Both questions aim at establishing a cycle, that is to say they both are definitorial questions for the cycles.[23]

Chronologically, these decompositions of the 532-year period were established by the 95-year Easter tables of Cyrillus of Alexandria and Dionysius Exiguus.[24]

There are still two other additive decomposition of the 532-year cycle:

$$532 = 4 \times 112 + 84$$
$$532 = 6 \times 84 + 28.$$

These equations do not belong to Alexandria, but to 2nd and 3rd century Rome. They link quite nicely together Greek thinking, pre-Christian and Christian. But, as they lie beyond the scope of this paper, I will not go into further detail here.

[23] Paul of Middelburg (1513) argued that the 532-year cycle can be established by the equation 532 = 95 + 437, while scholars, stubbornly insisting on Bede's "product of the cycle of the sun and of the moon", did not listen to him or did not understand him. U. Voigt, "Paul of Middelburg and the 437-year Period", in: *Proceedings of the 4th International Conference on the Science of Computus in Ireland and Europe*, Galway, 13–15 July, 2012, also www.likanas.de, p. 25 ff.

[24] U. Voigt, *Die 418-jährige Periode in der PRAEFATIO CYRILLI,* Teil I ("Alexandrinische Osterperioden und Ären"), www.likanas.de.

8. Lambert and Gauss

The algorithm published by Carl Friedrich Gauss (1800) [25] for the Easter date of a given year, does not answer the inverse problem:

To find the years to a given Easter date.

Now we have seen that the structure of the Great Easter Cycle only comes to light if the inverse Easter problem is taken into account. This is to say that Gauss did not reveal the true cyclical nature of the Easter date.

With my paper I want to do justice to the German mathematician Johann Heinrich Lambert who 25 years before Gauss published an algorithm that solves the inverse Easter problem for the Julian calendar. [26]

I do not know if Gauss knew of Lambert's publication, and decided to ignore it, though I have reason to believe that he did. In any case, my paper should have made clear that Lambert's treatment of the inverse Easter problem, far from being a mathematical gimmick, answered to an old and inevitable, though somewhat hidden question of Christian computus. Lambert, much like Gauss, was essentially a mathematician with only superficial concern for the historical background of his problems. In fact, he did not write a word about the relationship between the inverse Easter problem and the nature of the 532-year Easter cycle, and so it may well be that we have just brought him back down to earth.

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[25] Carl Friedrich Gauss, "Berechnung des Osterfestes", in: Zach, *Monatliche Correspondenz zur Beförderung der Erd- und Himmels-Kunde, August 1800,* Gotha 1800.

[26] Johann Heinrich Lambert, "Einige Anmerkungen über die Kirchenrechnung", in: *Astronomisches Jahrbuch oder Ephemeriden für das Jahr 1778 nebst einer Sammlung der neuesten in die astronomischen Wissenschaften einschlagenden Beobachtungen, Nachrichten, Bemerkungen und Abhandlungen*, Berlin 1776, 210-226.
A generalisation for the Gregorian case was published by the German historian Ferdinand Piper, "Zur Kirchenrechnung, Formeln und Tafeln", in: *Crelles Journal der reinen und angewandten Mathematik,* Bd. 22 (1841), 97–147.